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## STRUCTURAL SHAPE OPTIMIZATION IN MULTIDISCIPLINARY SYSTEM SYNTHESIS

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ABSTRACT. Structural shape optimization couples with other discipline optimization in the design of complex engineering systems. For instance, the wing structural weight and elastic deformations couple to aerodynamic loads and aircraft performance through drag. This coupling makes structural shape optimization a subtask in the overall vehicle synthesis. Decomposition methods for optimization and sensitivity analysis allow the specialized disciplinary methods to be used while the disciplines are temporarily decoupled, after which the interdisciplinary couplings are restored at the system level. Application of decomposition methods to structures-aerodynamics coupling in aircraft is outlined and illustrated with a numerical example of a transport aircraft. It is concluded that these methods may integrate structural and aerodynamic shape optimizations with the unified objective of the maximum aircraft performance.

### Nomenclature.

a -	subscript referring to aerodynamics.
c -	airfoil chord.
C -	cumulative constraint.
$e(Y_*, X)$ -	vector of equality constraints.
$F(X)$ -	system objective function.
$G(X)$ -	vector of system inequality constraints.
h -	airfoil depth.
$J_{as}$ -	$J(U_a, U_s)$ , jacobian matrix of partial derivatives of $U_a$ with respect to $U_s$ .
l -	subscript indicating lower bound.
M -	Mach number.
s -	subscript referring to structures.
u -	subscript indicating upper bound.
$U(X)$ -	vector of system behavior variables.
$U_i$ -	vector of behavior variables output from analysis of the i-th discipline.
X -	vector of system design variables.
$Y_i$ -	vector of i-th subsystem design variables.

### Introduction.

Structural optimization for shape has a long history of theoretical developments and successful applications, e.g., ref.1. It encompasses both the overall geometry of assembled structures, e.g., ref.2 (Ch.4 and 16), and the shape of individual components as in ref.1 (Ch.9). Its tool inventory includes a rich variety of methods ranging from the classical

variational approaches reviewed in ref.2 (Ch.5) to sophisticated finite element-based techniques, e.g., ref.1 (Ch.5 and 6). These developments have reached the point where it becomes practical for an engineer to use shape optimization methods not only in design of a structure treated as an isolated object, but also in design of structure whose shape determines its interaction with the environment.

Aircraft structural design deals with probably the most difficult cases where structures and the surrounding airflow interact through the shape of the structure boundary. Similar examples may also be drawn from naval architecture and automotive industry. Based on aeronautical experience, this paper presents the structural shape optimization as part of the overall aircraft system synthesis in which the structural discipline must interact with aerodynamics and other disciplines in search for the shape that maximizes the flight vehicle performance.

## **Structures - Aerodynamics Coupling.**

### **EXAMPLES OF COUPLING**

One mechanism that couples structures and aerodynamics in near-sonic cruise aircraft is the wave drag which causes the wing drag to rise as the flight Mach number approaches unity. The drag rise shown in Fig.1 shifts toward lower M and becomes steeper as the h/c ratio increases. On the other hand, the structural weight of a bending-dominated wing generally decreases with the increase of h/c. Consequently, a conventional shape optimization performed for minimum weight within the discipline of structures would tend to increase h/c, while an aerodynamic optimization for minimum drag at a constant M would tend to lower h/c. However, both weight and drag are detrimental to flight performance, such as payload delivered over a given range for a constant gross weight at take-off. Thus, a compromise between the two conflicting trends must be sought, by formulating a unified optimization problem, drawing its objective function from the system performance, and including constraints from aerodynamics and structures.

The optimization task is further complicated because the wing structural and aerodynamic behavior are coupled not only through the system performance but also directly via aerodynamic loads whose magnitude and distribution on the wing depend on the wing planform and airfoil shapes and on the wing structural deformations. Dependence of the deformations on the loads and wing shape completes the coupling.

Swept wings provide a second example of the effect of this type of coupling on the metallic wing structural weight plotted in Fig.1 as a function of sweep angle. Lift in a forward-swept wing increases the streamline airfoil angle of attack, that generates more lift, and so on - a positive feedback that may result in a wing divergence. A supercritical airfoil wing is a third example of the structures-aerodynamic coupling, again, through the loads and deformations. As shown in Fig.2, the supercritical airfoil's lift resultant is shifted aft relative to that of a conventional airfoil. As a result, torque on the wing box is increased and this may require more structural material for strength, and also for stiffness necessary to keep the wing from twisting excessively which redistributes the lift inboard with an attendant increase of induced drag.

### **METHODS FOR COUPLED, STRUCTURES - AERODYNAMIC OPTIMIZATION**

The structures-aerodynamics-performance interactions may be conveniently presented in the form of a directed graph (a means widely used in Operations Research (OR) system analysis, e.g., ref.3) shown in Fig.3. Each discipline on the graph represents a disciplinary analysis which

from a system's perspective is simply a black box transforming input into output. For the wing design, the inputs and outputs are inscribed by the arrows in the figure. Assuming that structure must fit inside of aerodynamic envelope, the external shape geometry is shown as an input from aerodynamics to structures. The loads and deformations add to the direct coupling of the two disciplines, the remainder of the coupling is through performance analysis. This representation of a wing as a system corresponds to the aspect decomposition defined in OR (ref.4) as one in which the system itself remains an indivisible object but each of its aspects (disciplines) is represented by a black box - as opposed to the object decomposition applicable when a black box representation may be assigned to each physically separable subsystems. Without decomposition, the structural shape optimization and aerodynamic design methods (e.g., ref.5) cannot be used to their full potential in design of a wing as a coupled system because neither design method accounts for the mutual influence of the two disciplines, and, therefore, the individual optimizations may work on cross-purposes. Further, optimization without decomposition requires repetition of the entire system analysis for every design variable perturbation (a multidimensional parametric study) to determine the full effect of the variables on the system performance. In contrast, the system decomposition approach temporarily decouples the disciplines and permits the application of specialized methods inside each black box. This is crucially important to being able to use the disciplinary advanced methods in design of coupled systems.

#### System Optimization Algorithms.

There are several ways the system optimization algorithm may be organized and the distinguishing element is usually the means of restoring the coupling to the temporarily decomposed system. Two recently developed algorithms are outlined.

#### HIERARCHAL, TOP-DOWN DECOMPOSITION.

When system optimization by hierarchal decomposition (ref.6, 7, and 8) is applied, the design variables are divided into the system variables that directly affect aerodynamics, structures, and performance, and the local (subsystem) variables which directly affect only aerodynamics, or structures. By these criteria, the wing planform and airfoil shape variables are system variables because they affect all three analyses. The structural weight is also a system variable since it enters the performance analysis. Amplitudes of basis functions representing the wing structural deformation are also variables at the system level due to the influence they have on aerodynamics and structural stiffness. In contrast, the structural cross-sectional dimensions are local variables affecting only the structure, and the variables defining the wing shape outside of the structural box are local variables affecting only aerodynamics.

The system optimization proceeds top down. The optimization problem solved at the system level in the i-th iteration is:

$$\begin{aligned}
 &\text{find } X, \text{ within move limits} \\
 &X_l \leq X \leq X_u, \\
 &\text{such that } F(X) \text{ is minimum, subject to constraints} \\
 &G(X) \leq 0, C_a \leq 0, C_s \leq 0.
 \end{aligned}
 \tag{1}$$

Examples of  $F(X)$  and  $G(X)$  are the take-off gross weight, and the required climb rate. The constraints  $C$  are cumulative constraints

formulated using the Kreisselmeier-Steinhauser envelope function (ref.9). They are differentiable functions of the constraints local to aerodynamics and structures, and approximate the largest (most violated) constraint providing a single measure of feasibility of each subsystem design. In eq. 1, the cumulative constraints are approximated as functions of X by linear extrapolation from which the approach takes its name. For example, the extrapolation for  $C_s$  is

$$C_s = C_{smin} + \nabla_x C_s \Delta X \quad (2)$$

The derivatives forming the  $\nabla_x C_s$  are carried from the aerodynamic and structural optimizations executed in the previous iteration. These subsystem optimizations may be generalized by a single definition, using \* as a substitute for a and s:

$$\begin{aligned} &\text{find } Y_* \text{ such that } C_*(Y_*, X) \text{ is minimum, subject to constraints} \\ &e(Y_*, X) = 0, Y_{*l} \leq Y_* \leq Y_{*u}; \end{aligned} \quad (3)$$

The purpose of the optimization is to proportion the wing shape and structure cross-sections to minimize the aerodynamic and structural constraint violations represented by C. The equality constraints, e, enforce compliance with the prescribed structural weight and structural deformation. The aerodynamic loads used in the optimization of the structure in the i-th iteration are computed in iteration i-1. The aerodynamic and structural optimizations are, in effect, decoupled and may be carried out concurrently. Optimum sensitivity analysis using the algorithm described in ref.10 or 11 yields derivatives of  $C_{*min}$  with respect to X to form the  $\nabla_x C_*$  used in eq.2. Thus, the shape design decisions made at the system level to improve the system performance include an approximate knowledge of their effects on aerodynamic and structural constraints.

#### NON-HIERARCHAL DECOMPOSITION.

Under this approach, decomposition is introduced only for the purposes of the system behavior sensitivity analysis while optimization is carried out for the entire system. The sensitivity analysis yields the derivatives used in linear extrapolations of the system behavior which substitute for system analysis in optimization performed in a piecewise-linear manner. The algorithm for sensitivity analysis by decomposition from ref.12 yields the derivatives of the system response as a solution of a set of simultaneous, linear, algebraic equations

$$\begin{bmatrix} I & -J_{as} \\ -J_{sa} & I \end{bmatrix} \begin{bmatrix} dU_a/dX_x \\ dU_s/dX_k \end{bmatrix} = \begin{bmatrix} \partial U_a / \partial x_k \\ \partial U_s / \partial x_k \end{bmatrix} \quad (4)$$

The Jacobians in the matrix of coefficients in eq. 4 represent sensitivity of the output from one disciplinary analysis to the inputs received from the other, e.g., partial derivatives of aerodynamic pressure with respect to the structural deformations. Similarly, the partial derivatives on the right hand side measure sensitivity of the output from aerodynamic and structural analyses to the design variables,

e.g., partial derivatives of structural deformation to the wing sweep angle as a shape variable. The partial derivatives, by definition, may be computed within each decoupled disciplinary analysis, so that specialized methods of disciplinary sensitivity analysis may be used, such as structural analysis for sensitivity with respect to shape, ref.1 (Ch.3), and the corresponding analysis in CFD formulated in ref.13. The method is new and experiences with its use in optimization is limited to ref. 14.

#### **Numerical Example.**

An optimization study in which hierarchal, top-down decomposition of the type outlined in the foregoing was applied to a transport aircraft shown in Fig.4 was reported in ref.15. The objective was to minimize fuel consumption subject to constraints on the aircraft performance, aerodynamics, and structures, including the detailed local buckling constraints of the stiffened panel in the wing covers. The design variables governed the airfoil shape and wing structure cross-sectional dimensions, down to the stringer detail level. Individual cover panel, ribs, and spars were represented in the finite element model. The procedure performance was satisfactory; a typical histogram for the fuel objective and structural weight is shown in Fig.5. It demonstrated that a large number of design variables (1303), including shape variables, and constraints (1950) representing diverse disciplines may be included in a large engineering system optimization, and that the system performance may be linked mathematically to the design details.

#### **Conclusions**

When shape optimization is applied to a structure interacting with the surrounding airflow, the optimization task becomes multidisciplinary and has to account for the coupling of structural mechanics and aerodynamics involving the mutual dependence of structural deformations and aerodynamic pressures. Aircraft design for optimum shape is an example of a large-scale engineering problem where the above occurs and where the shape optimization becomes a subtask in a system optimization because both structures and aerodynamics have a strong impact on performance.

Methods for system optimization by decomposition have been developed for such multidisciplinary applications. They allow use of specialized methods for optimization and sensitivity, by separating the disciplinary optimization tasks while accounting for the interdisciplinary couplings. As illustrated by a numerical example for a large transport aircraft, these methods have begun to approach the level of maturity required for effective applications in large scale engineering design.

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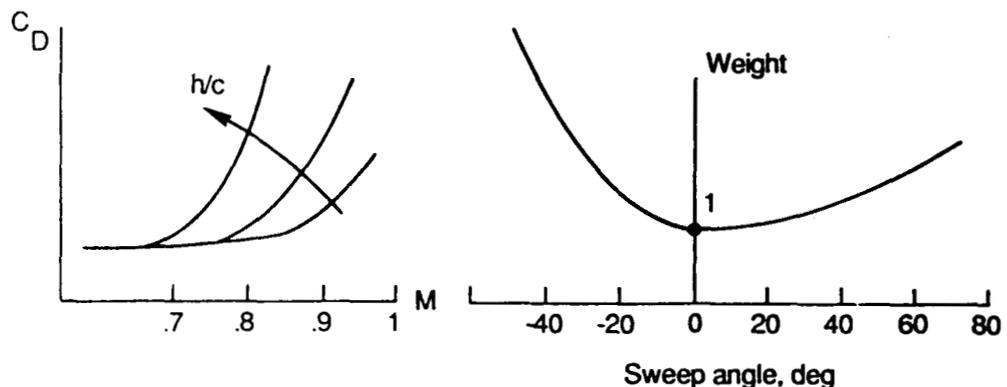


Figure 1. left- Drag rise as a function of  $M$  and  $h/c$ ; right- Structural weight of a wing vs. the wing sweep angle (positive = aft).

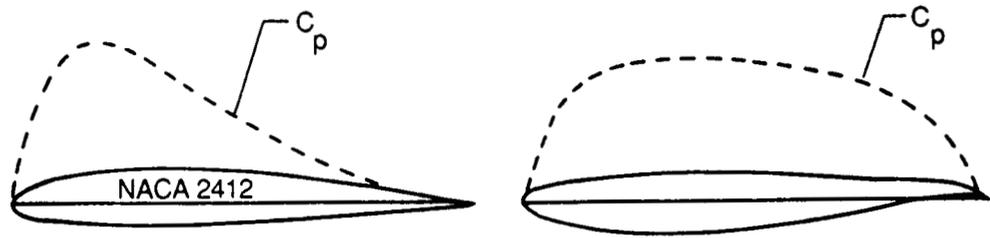


Figure 2. Aft shift of the lift resultant on supercritical airfoil (right) vs. conventional airfoil (left).

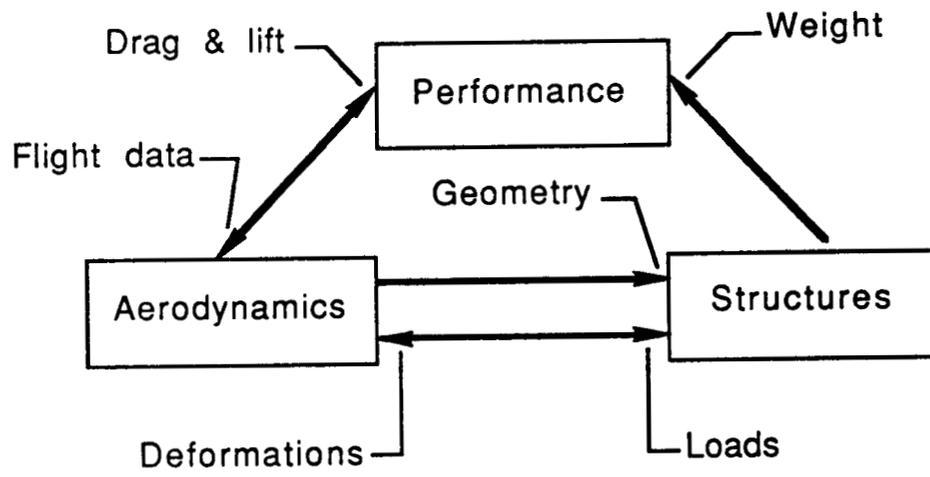


Figure 3. Disciplinary analyses coupled in wing design.

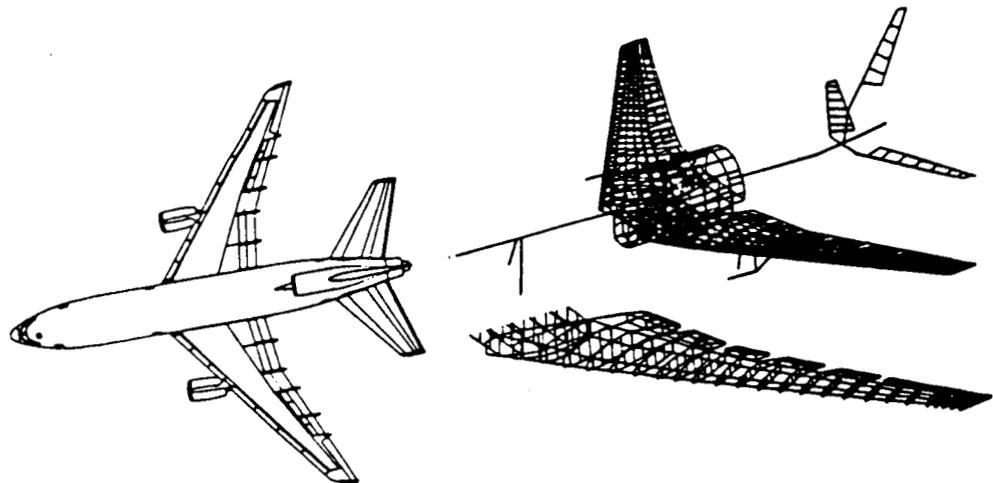


Figure 4. Transport aircraft wing optimization, ref 15.:aircraft, and its finite element model.

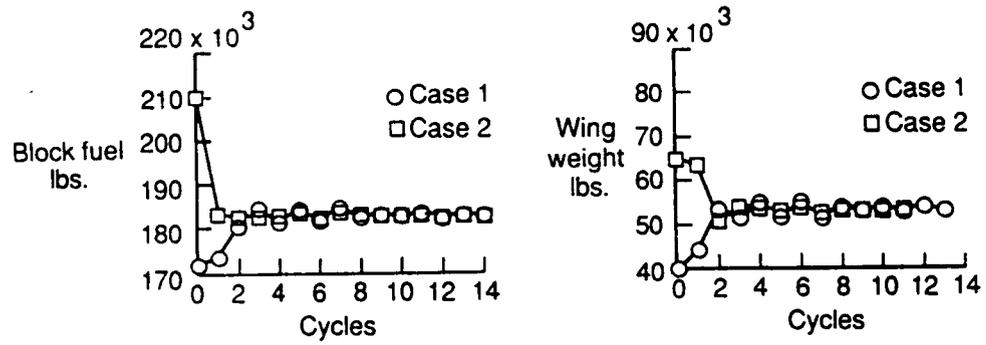


Figure 5. Histograms for the fuel consumption as objective function (left) and wing structural weight (right) for transport aircraft wing (case 1, 2: initial design infeasible and feasible, respectively).



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